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Integrated planning and allocation: A stochastic dynamic programming approach in container transportation

Guanghua Han[a], Xujin Pu[b,*], Zhou He[c], Cong Liu[d]

[a] School of International and Public Affairs, Shanghai Jiao Tong University, Shanghai 200030, China
[b] School of Business, Jiangnan University, Wuxi 214122, China
[c] School of Economics and Management, University of Chinese Academy of Sciences, Beijing 100190, China
[d] Beijing JD Shangke Information Technology Co.,Ltd., Beijing 100176, China

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ABSTRACT

Container transportation has developed rapidly in recent years because of the growth of international trade. However, transportation demands along shipping lanes or in different regions are unbalanced and change over time. This high-growth and uncertain operation environment makes empty container-capacity management important and challenging. Carriers usually face two types of demands from forwarders/shippers with long-term contracts and from the spot market. Empty container-capacity planning and allocation are based on demand information from forwarders and the spot market. In this paper, we focus on the empty container-quantity decision problem over one planning horizon of multiple schedules, each with a random demand. The carrier builds empty container capacity with its own containers and leased containers. We construct a stochastic dynamic program model to maximize the profit of the carrier. The objective function is shown to be concave in empty container quantity. We can also formulate a static model and a myopic model. We run simulations by assuming that demand follow colored and white Gaussian Noise processes, we observe that the optimal empty container quantity using the static model is identical or close to identical to that from the dynamic model, while the optimal empty container quantity from the myopic model is always more than that from the dynamic model. Therefore, a simplified iteration algorithm utilizing the static and myopic models is developed to obtain the optimal dynamic solution efficiently. Numerical experiments show that the proposed algorithm is effective.

1. Introduction

The uncertain demands, long facility-preparation time, high set-up costs, and rapidly changing market characterize the challenging operating environment for container carriers. Usually, a carrier leases a certain number of empty containers from container-leasing companies in addition to the containers it owns. In this challenging market environment, empty container management becomes one of the key areas affecting the competitiveness and profitability of the carrier.

Forwarders are closer to the market in shipping supply chains and possess private information on the shippers. Usually, a large proportion of the demand of a carrier is realized via forwarders (and directly with large shippers) based on long-term contracts, which come with high penalty costs when contract targets are not met. The remaining demand comes from the spot market, where orders are settled instantly at current market price. Since demands in spot market do not follow a set date, the spot market is also referred to as the 'cash market'. Huge of resources and efforts are payed to respond to uncertain instant demand from spot market, the corresponding price towards demand from spot market is relatively high. Although the spot market demand fluctuates considerably and creates volatility risk, it is usually quantity-limited than the contract demand. Therefore, carriers usually prioritize and satisfy contract demands first. Considering the high set-up cost and the relatively long lead-time, carriers should carefully plan empty container capacity for a shipping schedule before demand realization. Given the fixed number of self-owned empty containers, the empty container-capacity plan should focus on determining the number of empty containers to be leased from container-leasing companies.

In this paper, we consider a system with one carrier and two types of demands from forwarders (and large shippers) by contracts and from the spot market, or DF and DS, respectively. The carrier decides on the quantity of leased empty containers at the start of the shipping schedule. The shipping schedule can be divided into T schedules. In each schedule, both DS and DF are random, independent, and follow some stochastic distributions. The

* Corresponding author.
E-mail address: puyiwei@ustc.edu (X. Pu).

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carrier allocates empty containers to DS and DF at the start of each schedule in the shipping schedule. The planning processes include two stages, namely, the empty container-capacity preparation and the allocation. In the first stage, the carrier determines the total empty container capacity. In the second stage, the carrier allocates the containers to DS and DF in *T* schedules. The objective is to determine the optimal empty container quantity to maximize the carrier's total profits.

As the shipping networks become increasingly complex, the problem of the empty container assignment has become more troublesome. Several studies focus on empty container-preparation and assignment problems. Dejax and Crainic [1] provide a comprehensive review on the works dedicated to the container transportation aspects of the problem. Cheung and Chen [2] present a two-stage stochastic network model for dynamic empty container allocation and repositioning over time and among ports along a shipping lane, while assuming unlimited availability of leased containers at every port. Li et al. [3,4] propose a policy of controlling the quantity of company-owned and leased containers for a single port, and extend their model by introducing empty container repositioning and allocation for multi-ports. Allocation problem was formulated and heuristic methods were designed according to the average cost using the (u, d) policy at one port. Francesco et al. [5] study a container maritime-repositioning problem where several parameters were uncertain and historical data could not be used for decision-making process. Most of the above works focus on the internal resource optimization of the carrier and does not consider the external coordination of the empty container-assignment problem. We devote ourselves to the container assignment problem of the carrier in the container shipping service chain, which includes one upstream rental company, one carrier, one forwarder, and the spot market.

Some researchers have studied the external coordination between participants in the container shipping service chain to optimize empty container quantity. Caplice [6] discusses the request-for-proposal procurement process for transportation services. He believes that the predominant form of commercial relationship between shippers and carriers has changed from a transactional basis to a contractual one. Schönberger [7] constructs a model for collaboration among transportation companies and provides a memetic algorithm to solve the problem. Bu et al. [8] analyze the contract decision problem of the marine shipping capacity option, and establish an optimal decision model of the carriers and the forwarders for shipping capacity contract with empty container-repositioning cost.

To summarize, the related existing research can be divided into two streams, namely, the allocation plans and the coordination between participants to optimize the container capacity. No research has been made on the integration of empty container-preparation and allocation decisions. In addition, the existing study mainly considers demands from one source (the forwarder or the shipping market). In this paper, we consider the demands from both forwarders and the spot market. The contributions of this paper can be summarized as follows. First, this paper studies a realistic integrated container preparation and allocation problem with two types of random demands and multi-allocation schedules. Second, this paper models the problem by using a stochastic dynamic method, and develops a simplified algorithm to reduce the computational burden.

The rest of the paper is organized as follows. Section 2 describes the basic model of a stochastic dynamic programming model. Section 3 shows the proof that the objective function is concave in the optimal rented empty container quantity, presents the available allocation framework, and shows that the dynamic allocation policy is the optimal policy for allocating empty containers. Section 4 presents an effective iterative algorithm designed to solve the proposed stochastic dynamic programming model and shows a numerical example. Section 5 concludes the paper.

2. The model

The shipping system includes one carrier, one forwarder, and a spot market. Two streams of demands, the DF and the DS, are available. To satisfy the demands, the carrier has to decide the number of empty containers available for each demand stream at the start of a shipping schedule. In the container shipping industry, carriers often lease most of their containers from leasing companies. A leasing schedule usually lasts from three to six months. For each leasing schedule, the carrier schedules several shipments for the leased containers such that the leasing schedule can be divided into a number of schedules with each corresponding to a shipment voyage. In this paper, we focus on one leasing schedule with multiple shipping schedules. We assume that the carrier makes its decision in two stages. Fig. 1 shows the empty container-capacity preparation stage or stage 1, and the allocation stage is stage 2. In stage 1, the carrier decides on the total number of empty containers available for the leasing schedule. In stage 2, the carrier allocates dynamically the empty containers to the two types of demands in each of the *T* schedules. In each schedule, the DF and the DS are random and independent of each other and follow some stochastic distributions. In addition, we assume that one leasing schedule contains a number of voyages. The first voyage returns to the originating port at the end of the leasing schedule; thus, no empty containers can be brought back by the voyages in this leasing schedule. The objective is to maximize the total profit of the carrier by deciding the optimal empty container quantity at the beginning of the leasing schedule and the optimal dynamic allocation policy. Table 1 lists the notations we use in this paper.

This study adopts the following assumptions:

Assumption 1. *The demands from the spot market and the forwarder in each schedule are random with known probability density function.*

Assumption 2. *The capacity salvage value is zero, and all unsatisfied demands are lost.*

The carrier initially owns K_0 empty containers. The number of rented empty containers from the leasing market is denoted by Q_0.

Stage 1. Empty container-capacity decision stage

The number of leased empty containers Q_0 is determined. The leasing cost of each unit is c_0 for the whole leasing schedule. The total number of empty containers available for the leasing schedule is $Q = Q_0 + K_0$. In industry practice, a carrier first satisfies the demands from its downward forwarders under long-term contracts and then allocates a quantity of empty containers to the spot market.

Stage 2. Container allocation stage

This stage consists of *T* schedules. Demands $d^t = (d_F^t, d_S^t)$ from the spot market and the forwarder at schedule *t* are observed at the beginning of *t*. Let X^t be the number of available empty containers at the beginning of schedule *t*. Hence, $X^1 = Q$. Let N^t be the difference between the available empty containers x^t and the actual demands d_F^t in the current schedule, that is, $N^t = (x^t - d_F^t)$.

N^t can either be positive, negative, or zero. The allocation decisions made in schedule *t* are based on both N^t and the demands in the next schedules. Considering that the demand from the spot market $d_S^t \geq 0$ must be realized, the carrier can decide the quantity of the demand to be satisfied. Let the allocated quantity be y_S^t, which is positive and within the current capacity of the available empty containers.

$$0 \leq y_S^t \leq \min((N^t)^+, d_S^t).$$

The carrier decides on the allocation at the beginning of each schedule after the demands are realized. The excess empty con-

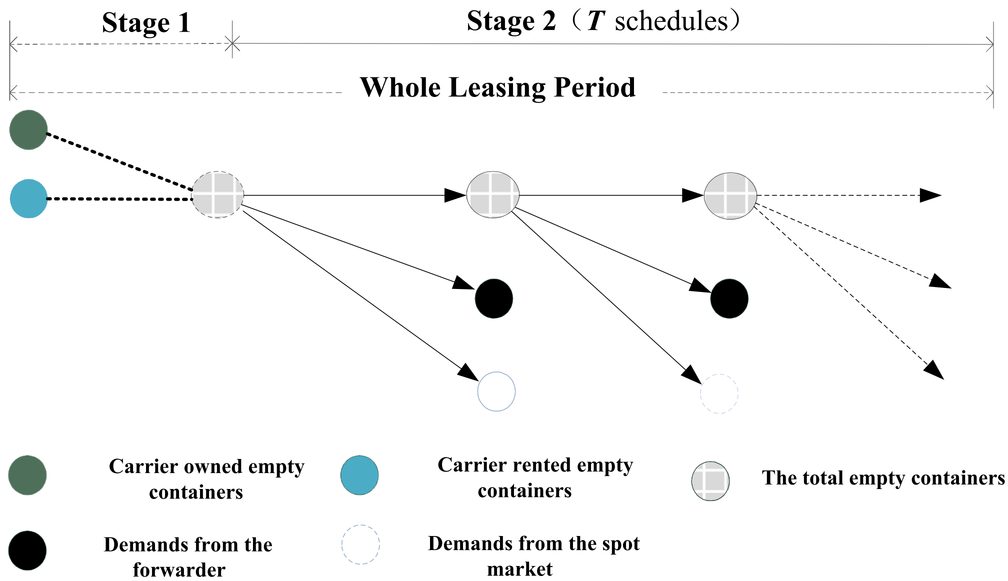

Fig. 1. Two stages of the whole time horizon.

Table 1
Notations.

T	Total schedules in the container allocation stage
K_0	Quantity of carrier-owned empty containers
Q_0	Quantity of empty containers rented by the carrier from the leasing market
Q	Total number of empty containers available for the whole leasing schedule; thus, $Q = K_0 + Q_0$
c_0	Rental cost of each empty container
D^t	Demands for empty containers at schedule t, $D^t = \{D_S^t, D_F^t\}$. The realized values are $d^t = \{d_S^t, d_F^t\}$, where d_F^t is the realized DF at schedule t and d_S^t is the realized DS at schedule t
Y^t	Allocated empty containers at schedule t. The realized values are y_S^t (containers allocated to the spot market) and y_F^t (containers allocated to the forwarder)
X^t	Available empty containers at the start of schedule t. The realized value is x^t
N^t	Difference between the actual demands d_F^t and available empty containers x^tat the current schedule, that is, $N^t = (x^t - d_F^t)$
p_F	Unit price of empty containers sold to the forwarder
p_S	Unit price of empty containers sold to the spot market
v_F	Penalty cost when one DF unit is not fulfilled
v_S	Penalty cost when one DS unit is not fulfilled
u_F	Processing cost of satisfying one DF unit
u_S	Processing cost of satisfying one DS unit
w	Shipping cost of each container
α_F	Contribution margin from satisfying one DF unit
α_S	Contribution margin from satisfying one DS unit
$\bar{\alpha}_S$	Contribution margin of empty containers when some empty containers are reserved for the next schedule
$\Pi(Q_0)$	Total profit of the carrier for the whole service time horizon
$\Pi^t(X^t)$	Total profit in $T - t + 1$ schedules (from schedule t to schedule T)
	Revenue at schedule t
$H^t(Y^t/D^t)$	
Q_0^{M*}	Optimal quantity of prepared empty containers using the dynamic policy
Q_0^{M*}	Optimal quantity of prepared empty containers using the myopic allocation policy
Q_0^{S*}	Optimal quantity of prepared empty containers using the static policy

tainer capacity at the end of schedule T has no salvage value. Therefore, the cost for the time horizon contains two parts, which are the container leasing cost and the penalty cost from the unsatisfied demand. The revenue comes in the allocation stage. Let $\Pi(Q_0)$ be the profit function for the whole leasing schedule. Our objective is to find the optimal leased empty container quantity Q_0 and allocations for the T schedules to maximize this profit function. We formulate this problem as a dynamic program with $T+1$ steps. In the empty container-preparation stage, the carrier determines the optimal rented container quantity, whereas in schedules 1 through T, the carrier allocates its empty containers to maximize its revenue. Let p_F denote the price of satisfying one DF unit and v_F denote the unit penalty cost when one DF unit is not fulfilled. Let u_F denote the processing cost (search for the demand, activities on the order, and so on) for each DF, and u_S be the processing cost for each DS. The processing cost of each DF is smaller than that of each DS because of the closer relationship between the carrier and the forwarder and a bigger volume, which is,

$$u_F \le u_S \tag{1}$$

Let α_F be the contribution margin for satisfying a DF unit, and let α_S be the contribution margin for satisfying a DS unit. Let w be the shipping cost for each container. Clearly, the contribution margins are $\alpha_F = p_F + v_F - u_F - w$ and $\alpha_S = p_S + v_S - u_S - w$. $p_F + v_F$ and $p_S + v_S$ represent the carrier revenue from satisfying one unit each of DF and DS, respectively. $v_F > v_S$ due to the relatively bigger punishment for an unsatisfied DF. The contribution margin of one satisfied DF is more than that of DS, that is, $\alpha_F > \alpha_S$ (for example, see [10]).

As commonly practiced in the shipping industry, we assume that the carrier satisfies first the demands of the forwarder. The remaining empty containers are then used to satisfy the demands of the spot market. In addition, a discounted price is applied to the demands of the forwarders, which is a common practice in the shipping industry. Thus, the following relationships hold:

$$u_F \le u_S,\ v_F > v_S,\ p_F < p_S,\ \alpha_F > \alpha_S \tag{2}$$

The dynamic model can now be formulated as follows:

Stage 1: Leased empty container-quantity decision stage

$$\Pi(Q_0) = \max_{Q_0}\{\Pi^1(X^1) - c_0 Q_0\} \tag{3}$$

where

$$X^1 = Q_0 + K_0 \tag{4}$$

Stage 2: Allocation stage ($1 \le t \le T$)

$$\Pi^t(X^t) = \underset{D^t}{E}\left\{\max_{Y^t + X^{t+1} = X^t}[H^t(Y^t/D^t) + \Pi^{t+1}(X^{t+1})]\right\} \tag{5}$$

$$H^t(Y^t/D^t) = \max_{y_F^t, y_S^t}[\alpha_F y_F^t + \alpha_S y_S^t - v_F d_F^t - v_S d_S^t] \tag{6}$$

$$s.t.\quad y_S^t \le d_S^t \tag{7}$$

$$y_F^t \le d_F^t \tag{8}$$

$$y_F^t + y_S^t \le X^t \tag{9}$$

$$y_F^t, y_S^t, Y^t, X^{t+1}, Q_0 \in R^+ \tag{10}$$

$\Pi^t(X^t)$ at the left-hand side of Eq. (5) denotes the total profit of the $T - t + 1$ schedules (from schedule t to schedule T) given available empty container X^t. Similarly, $\Pi^1(X^1)$ in Eq. (3) is the total profit of the entire allocation stage. Thus, Eq. (3) maximizes the total profit for the whole leasing schedule, that is, the total profit in the allocation stage minus the cost in the empty container-preparation stage. $\Pi^{T+1}(X^{T+1})$ is equal to zero because the salvage value is zero. Eq. (4) describes the total empty containers at the beginning of the allocation stage. Y^t denotes the number of available empty containers allocated in schedule t. Thus, $Y^t + X^{t+1} = X^t$ ensures that the sum of the containers used in schedule t and the containers held over for the next schedule is equal to the number of available containers at the beginning of schedule t. Eq. (5) states that $\Pi^t(X^t)$ is the sum of the profit in schedule t and the profit obtained in the next schedules. y_S^t and y_F^t are the satisfied DS and DF, respectively. $H^t(Y^t/D^t)$ in Eq. (6) is the profit for a single schedule t, given realized demand $D^t = (d_N^t, d_S^t)$. Eqs. (7) to (9) are the demand constraints at schedule t. Eq. (10) shows that variables $y_{i,j}^t, Y^t, X^{t+1}$, and Q_0 are positive numbers. The revenue at schedule $T+1$ and the future schedules are zero, that is, $\Pi^{T+1}(X^{T+1}) = 0$ because the empty containers have no salvage value. As the $\Pi^1(X^1)$ value is the accumulative value from schedule T to schedule 1, the computation of $\Pi^1(X^1)$ creates a dynamic stochastic programming model with T steps.

3. Model analysis

The allocation decision depends not only on the demand and the available empty container in the current schedule, but also on the demands in future schedules. Future demands are uncertain and are independent of each other; thus, the dynamic model $\Pi(Q_0)$ is a $(T+1)$-step stochastic dynamic programming model.

Lemma 1. $\Pi(Q_0)$ is concave in Q_0.

Proof. $\Pi^{T+1}(X^{T+1}) = 0$ because $\alpha_F > \alpha_S$ and the salvage value is zero. From Eq. (5), we have

$$\Pi^T(X^T) = \underset{D^T}{E}\{\max[H^T(X^T/D^T)]\} \tag{11}$$

$$s.t.\quad y_S^T \le d_S^T,\ y_F^T \le d_F^T \tag{12}$$

$$y_F^T + y_S^T \le X^T \tag{13}$$

$$y_F^T, y_S^T, X^T \in R^+. \tag{14}$$

Eq. (6) shows that $H^T(Y^T/D^T)$ is concave in X^T. Therefore, $\Pi^T(X^T)$ is concave in X^T because expectation preserves concavity (Van Slyke and Wets [9]).

Now, we assume that $\Pi^{t+1}(X^{t+1})$ is concave in X^{t+1}. Obviously, $Y^t = (y_F^t, y_S^t)$ determines the right-hand-side of the constraints in $H^t(Y^t/D^t)$. Thus, function $H^t(Y^t/D^t)$ is concave in Y^t. The relationship $Y^t + X^{t+1} = X^t$ reveals that $\Pi^t(X^t)$ is concave in X^t, which is the maximum value of the sum of two concave functions.

The above analysis shows that $\Pi^1(X^1)$ is concave in X^1. $X^1 = Q_0 + K_0$ is a positive linear function in Q_0. Thus, $\Pi^1(X^1)$ is concave in Q_0. Obviously, $-c_0 Q_0$ is a linear function in Q_0; thus, $\Pi(Q_0) = \Pi^1(X^1) - c_0 Q_0$ must be concave. *Q.E.D.*

The contribution margins for satisfying one unit each of DF and DS are $\alpha_F = p_F + v_F - u_F$ and $\alpha_S = p_S + v_S - u_S$, respectively. In $\alpha_F > \alpha_S$ (See Eq. (2)), the carrier first satisfies the DF before satisfying the DS. Therefore, the allocation solutions at schedule t are

$$y_F^t = \min(d_F^t, X^t),\ 0 \le y_S^t \le \min((X^t - y_F^t), |N^t|) \tag{15}$$

Next, we assume that the optimal container quantity allocated to DS at schedule t is y_S^t, and the contribution margin of the last unit container allocated to DS is α_S. However, if some empty containers are reserved for the next schedule, the contribution margin of the empty containers is

$$\begin{aligned}\bar{\alpha}_S(y_S^t) &= \Pi^{t+1}(X^{t+1} + 1) - \Pi^{t+1}(X^{t+1}) \\ &= \Pi^{t+1}(X^t - y_F^t - y_S^t + 1) - \Pi^{t+1}(X^t - y_F^t - y_S^t).\end{aligned} \tag{16}$$

where $X^t \ge 0$

While the reserved empty containers are allocated to the DF or DS in future schedules, they may not be allocated (in the worst case scenario) in future schedules due to smaller-than-expected demands in the future schedules. Thus, $0 \le \bar{\alpha}_S(y_S^t) \le \alpha_F$.

Because $\Pi^{t+1}(X^{t+1})$ is concave in X^{t+1} (See Lemma 1), $\bar{\alpha}_S(y_S^t)$ increases in X^t. Given that α_S is a constant parameter, the optimal container quantity allocated to DS is the maximum of all possible y_S^t values that satisfies $\bar{\alpha}_S(y_S^t) \le \alpha_S$. However, if no y_S^t value satisfies the inequality $\bar{\alpha}_S \le \alpha_S$, the marginal profit for satisfying the DS is always less than that of reserving the container for the next schedule. Therefore, the optimal allocated container to the spot market is zero. In short, the optimal allocated quantity is summarized as

$$y_S^{t*} = \begin{cases}\max(y_S^t), & if\ \bar{\alpha}_S(y_S^t) \le \alpha_S \\ 0, & else.\end{cases} \tag{17}$$

Remarks. *Influences of noise and interconversion in demand to problem formulation and allocation policy.*

Both Eqs. (15–17) suggest the optimal empty container allocation policies respecting to the demands from spot market and forwarders. In detail, allocate as more as possible to the demand from forwarders, and allocate the most appropriate amount of empty containers to spot market basing on a deterministic process. We do not emphasize the relations between the DF demand and DS demand when we buildup the decision model by dynamic model (specified by Eqs. 3–10) and come up to the allocation policies. Specifically, demand may follow different sorts of stochastic processes over time, like white or colored noise of demands, the interconversion of DF demand and DS demand. One essential question to answer is whether the model and allocation policy robust to stochastics of demands. We answer the questions in the following two aspects.

First, we examine the influence of stochastics of demand to problem formulation. Since the decision maker determines its capacity of empty container and its allocation quantity in different time schedule, the decision problem follows a stochastic dynamic process, which is thereby formulated as a stochastic dynamic programming. In other words, the dynamic program suggests a step-by-step decision processes, the optimal decision is made based on the realized demand in current schedule and the unrealized demand with given distribution in forthcoming schedules. Since the demand distribution is able to be known under different circumstances, e.g., noise in demand or alternativeness between two types of demand, the formulated model still holds and is robust with different shapes of demands.

Second, the robust of allocation policy. Because the contribution margin of satisfying DF demand is always than that of satisfying DS demand in current schedule and demand in upcoming schedule, the optimal allocation policy is allocating as more as empty containers to satisfy DF demand. Obviously, how much empty container to allocate is determined by the realized value of DF demand, where DF demand follows a probability density function. Meanwhile, we compare the contribution margin of satisfying DF demand on current schedule and of that when reserve the empty container to next schedules, where the contribution margin of reserved empty container is determined based on the DF and DS demands with known distributions. Thus, Eq. 17 specifies the allocation decision to satisfy DS demand and works for different known distributions of DF and DS demands.

Since the dynamic programming model and allocation policies work for demands with all kinds of shapes, the shape factors, e.g., noise of uncertain demand as well as interconversion in demands, does not affect the forms of the model and empty container allocation policies.

4. Simplified algorithm

Demands are realized over schedules, and the allocation decisions should be made at the beginning of each schedule. Considering that the profit from satisfying the DS is lower than satisfying the DF, the decision of satisfying the DF at each schedule is made not only based on the current demand but also on future demands (DF and DS), that is, the DS in the current schedule will not necessarily be fully satisfied. The allocation rule that considers the future demand is called dynamic policy, and the corresponding model (see Eq. (3)) is called the dynamic model.

To obtain the optimal solution of the dynamic model, the allocation strategy should be based on Eq. (17). Therefore, the allocation decisions in each schedule are made not only based on the current available empty containers and demand, but also on the demands in the future schedules. Whereas Eq. (17) gives the optimal allocation policy, the calculations for the optimal allocated quantities are burdensome due to the unrealized demands in Eq. (17).

The optimal number of leased containers can be searched numerically by iteration because the profit of the carrier is concave in the quantity of leased containers. If the optimal leased empty container quantity is Q in iteration step w, at least $2+Q/w$ times the stochastic dynamic programming should be calculated. Thus, searching for the optimal quantity of the leased container represents a significant computational burden. The situation worsens when the number of schedules increases. In this section, we develop a new iterative search method to speed up the search for the optimal empty container quantity.

4.1. Model comparisons

Two other simpler allocation policies available are myopic and static. Future demands are not considered in the allocation in the current schedule under the myopic policy. If we assume $N^t>0$, the satisfied DS is $y_S^{D\_t}$, and the satisfied DS using the myopic policy is $y_S^{M\_t}=\min(N^t,d_S^t)$. Obviously, the satisfied DS from the myopic policy is more than the result of the dynamic policy.

Determining the optimal allocation decisions is difficult because demands that arrive in each future schedule are unknown. However, if all demands are known before the allocation stage, the problem will become an optimal empty container-allocation problem with perfect information. The optimal empty container-capacity allocation model with perfect information (called the static model) is described as follows:

Empty container-preparation stage:

$$\Pi^S(Q_0^S)=\max_{Q_0^S}\{\Pi^{S\_1}(X^{S\_1})-c_0Q_0^S\} \tag{18}$$

$$X^{S\_1}=Q_0^S+K_0^S \tag{19}$$

Allocation stage ($1\le t\le T$):

$$\Pi^{S\_1}(X^{S\_1})=\max_{y_S^{S\_t},y_F^{S\_t}}\ \mathop{E}_{d_S^t,d_F^t}\left(\sum_t\alpha_F y_F^{S\_t}+\sum_t\alpha_S y_S^{S\_t}-\sum_S\upsilon_S d_S^t - \sum_F\upsilon_F d_F^t\right) \tag{20}$$

$$s.t.\quad y_S^{S\_t}\le d_S^t,\ y_F^{S\_t}\le d_F^t \tag{21}$$

$$y_F^{S\_t}+y_S^{S\_t}\le X^{S\_t},\ y_S^{S\_t}+y_F^{S\_t}+X^{S\_t+1}=X^{S\_t} \tag{22}$$

$$y_F^{S\_t},y_S^{S\_t},Y^{S\_t},X^{S\_t+1},Q_0^S\in R^+$$

$\Pi^S(Q_0^S)$ in Eq. (18) includes two parts: the total profit in the allocation stage and the total cost when the carrier rents the containers. As all demands are realized at the beginning of the allocation stage, the allocation problem can be transformed into a single-schedule decision-making model. Let $y_S^{S\_t}$ and $y_F^{S\_t}$ be the empty container allocation for demands, d_S^t and d_F^t, respectively, at schedule t. Eq. (20) is the total profit obtained at the whole empty container-allocation stage. Eqs. (21) and (22) are the demand and supply constraints, respectively. $y_F^{S\_t},y_S^{S\_t},Y^{S\_t},X^{S\_t+1}$ and Q_0^S are non-negative numbers.

The satisfied demand by the myopic allocation policy is at least as many as that by the dynamic allocation policy from Eq. (15). Future demand information is not needed by the static allocation policy. Hence, fewer calculations are needed for the static allocation policy than that for the dynamic allocation policy. Obviously, the dynamic policy is more profitable than the myopic policy (Appendix 1). This conclusion is critical for the search of a simulation-based allocation policy.

Once a certain empty container capacity is given, the satisfied DF quantities both by the myopic and dynamic policies are not always the same, and the dynamic allocation policy is more profitable than the myopic allocation policy. We designed a simulation-based experiment to test the quantity difference by using the two allocation policies, and important observations are drawn.

Observation 1. *The optimal quantity of the empty containers under the dynamic policy is not more than the myopic policy.*

Observation 2. *The optimal quantity of empty containers under the dynamic policy is almost identical to the static policy.*

The analysis of the model shows that the dynamic policy is more profitable than the myopic policy because of better flexibility. Let Q_0^{D*} be the optimal empty container quantity under the dynamic policy and Q_0^{M*} and Q_0^{S*} be the optimal empty container quantities under the myopic and static allocation policies, respectively. In the following context, two experiments using the design of experiment (DOE) method is adopted to demonstrate the theoretical analysis results of Observations 1 and 2.

Design of Experiments

In many industrial practices, demands are dependent over time. For example, in shipping market, demands for transportation have seasonality pattern, demand for empty container grows in selling season but decreases in after the selling season [14]. As well, demand from different resources has its internal characteristics which is alterable over time. In container freight market, the consumers' demand from spot market (i.e., DS demand) turns into the demand from the forwarder market (i.e., DF demand) when the he/she employ a forwarder to deal with his/her demands for empty container. Similarly, when a consumer does not rely on the forwarders' service and trade instantly with freight company, the DF demand transfers to DS demand. The variation of DF demand Δd_F^i and DS demand Δd_S^i because of transformation of demand is able to be calculated by a simple mathematical formula, $\Delta D_F^i = -\alpha d_F^i + \beta d_S^i$ and $\Delta D_S^i = \alpha d_F^i - \beta d_S^i$, where α and β are constant coefficients representing the proportion of transferred demands. Thus, α and β are ranges in [0,1]. When a coefficient equals to zero, the type of demand does not transfer to the other. On contrary, if a coefficient equals to 1, all of the type of demand transfers to the other type.

In container transportation industry, demand for empty container always time-dependent. For example, according to data from Drewry, the third quarter of the year is traditionally the busiest for the container shipping industry as retailers stock up for the holiday season [15]. Since demands in different schedule are time-oriented, we assume that both DF demand and DS demand follow colored Gaussian noise processes in the experiment. Thus, we let X_F^i and Y_S^i are variables follows random distributions, we have $d_F^{i+1} = \varepsilon + \in X_F^i$ and $d_S^{i+1} = \theta + \vartheta Y_S^i$, where $d_F^i \sim X_F^i$, $d_S^i \sim Y_S^i$ and $\varepsilon, \in, \theta, \vartheta$ are exogenous constants. Thus, we have uncertain demands DF and DS under discrete-time additive white Gaussian noise when $\varepsilon = \theta = 0$ and $\in = \vartheta = 1$, the assumption is used in many existing studies [11–13], etc. Since uncertain demand is able to be time-dependent or time-independent, we analyze decisions considering different types of demand. In the first experiment, we assume DF and DS demands are time-series with colored Gaussian noise. In the second experiment, we assume both DF demand and DS demand follow normal distributions with white Gaussian noise. We examine Observations 1 and 2 by conducting two experiments with different type of Gaussian noise.

Experiment 1. Correlative Demands with Colored Gaussian Noise

We conduct the experiment of container capacity preparation and allocation problem with two schedules, and compare the solutions between different allocations polices. Since myopic policy and static policy are more simple and straightforward to apply in practice, the solution processes could be simplified if the advantages of the two polices is employed in solving our problem. Considering the substitutive feature between DF and DS demands and time-series characteristic of demand in multiple schedules, we assume DF demand and DS demand are two-way transferred and follow colored Gaussian noise processes. Employing the assumptions of the uncertain DF and DS demands, we conduct the experiment under a two-schedule allocation problem and compare the solutions with dynamic, myopic and static allocation policies. The parameter values are assumed as follows. >

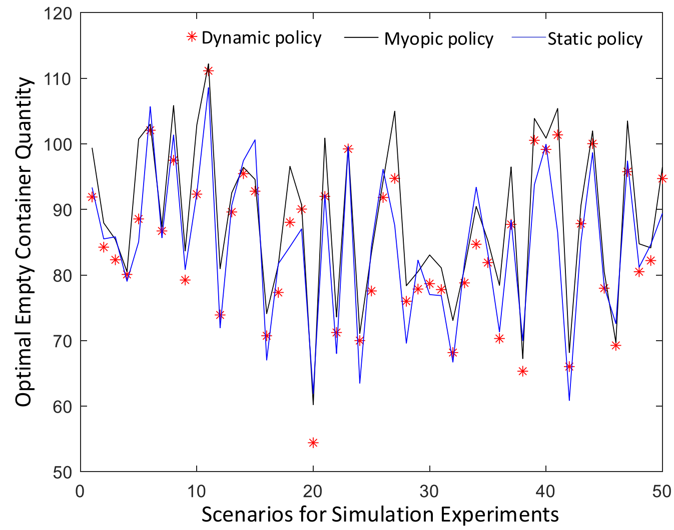

Fig. 2. Optimal empty container quantity by different allocation policies.

$$d_F^2 = (1-\alpha)\left(\varepsilon + \in X_F^1\right) + \beta\left(\theta + \vartheta Y_S^1\right),\ d_S^2 = \alpha\left(\varepsilon + \in X_F^1\right) + (1-\beta)\left(\theta + \vartheta Y_S^1\right),\ d_F^1 \sim X_F^1,\ d_S^1 \sim Y_S^1$$

where $X_F^1 = n(30, 16)$, $X_S^1 = n(28, 12)$, $\alpha = 0.15$, $\beta = 0.1$, $\varepsilon = 10$, $\theta = 5$, $\in = 0.0.96$, $\vartheta = 0.92$

Since the time-series feature of DF demand and DS demand, we calculate the Hurst exponents of two type of demand. We find the Hurst exponents of DF demand is 0.72 and that of DS demand is 0.68, which means DF and DS demands have strong long-term persistence. Meanwhile, penalty cost, carrier-owned container quantity, and empty-container preparation cost follow normal distribution (truncated at zero and positive). That is, $c_0 \sim n(6, 12)$, $v_1 \sim n(6, 11)$, $p_1 \sim n(30, 11)$, $u_1 \sim n(7, 10)$, $K_0 \sim n(10, 16)$, and $w = p_1 B(3, 5)$, where n denotes normal distribution and B denotes beta distribution. The revenue for satisfying one DF unit is $\alpha_F = p_1 + v_1 - u_1 - w$. According to both real practice and our assumptions, $\alpha_F > \alpha_S$. Thus, we let $\alpha_S = \alpha_F B(5, 8)$. To perform the comparative study, the values of each parameter in the three allocation policies are assumed as identical. We generate 50 scenarios to compare the optimal empty container quantity by using the different allocation policies (Fig. 2).

We show the simulation experiment results on the optimal empty container quantity from the three allocation policies by Fig. 2. The experiment with 50 scenarios shows that the optimal empty container quantity using the static allocation policy is not always more than the quantity resulting from the myopic policy or lesser than the quantity resulting from the dynamic policy. However, the optimal empty container quantity using the myopic policy is always more than or equal to that using the dynamic policy. The results provide directions for effectively searching the solution to the optimal input material quantity of the system under its upper bound which suggested by the solutions under myopic policy. To confirm this observation, more scenarios are generated by the DOE method. A two-tailed test statistical experiment is designed, and the corresponding confidence level, test power, and permissible error of the result are 0.95, 0.1, and 1, respectively. We generate 10,000 random scenarios such that more extreme values and parameter combinations are included in the experiment. The value of $Q_0^{M*} - Q_0^{D*}$ for each scenario are plotted in Fig. 3, we have the mean of $Q_0^{M*} - Q_0^{D*}$ values is 4.2888, and the standard variance is 3.2115; hence, the basic effective scenario size fora robust evalua-

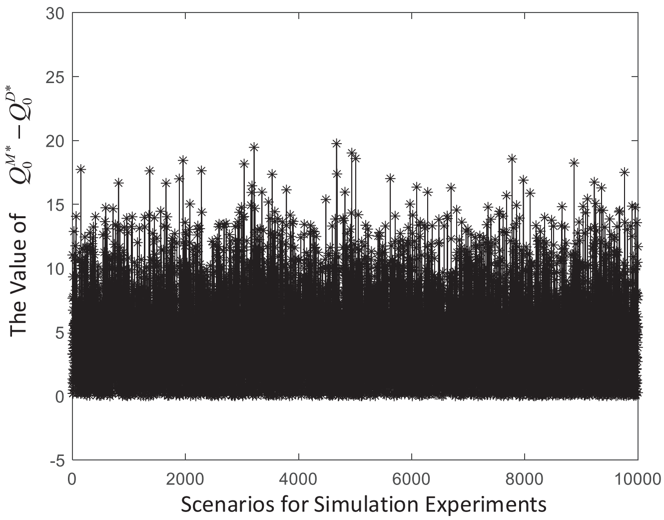

Fig. 3. Comparison of optimal empty container quantities using the myopic and dynamic policies with big sample sizes.

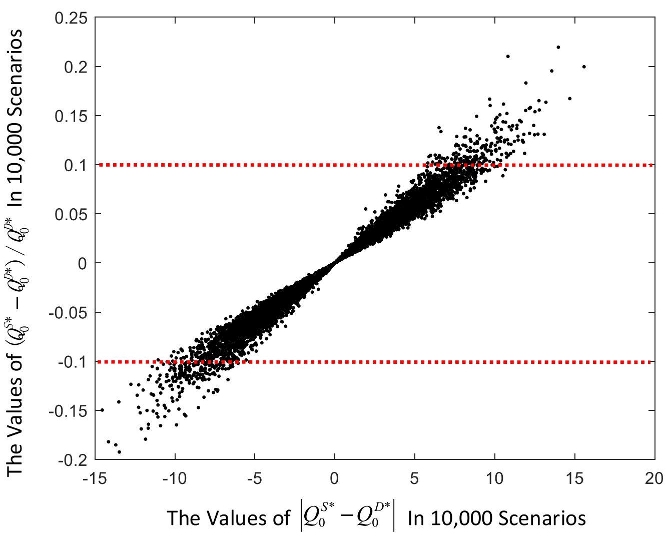

Fig. 4. $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ values in 10,000 scenarios.

tion is just 7,038 by using the DOE theory. In other words, we generate an adequate number of scenarios to perform the experiment effectively.

In order to verify observation 2, we test the $(Q_0^{M*} - Q_0^{D*})/Q_0^{D*}$ values for 10,000 scenarios, only 30.81% have corresponding values within the interval [−0.1, 0.1]. However, the values of $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ and $(Q_0^{M*} - Q_0^{D*})/Q_0^{D*}$ from the 5,000 simulation scenarios are also computable. Comparing the values of $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ in each scenario, we observe 9,183 scenarios (91.83%) have $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ values within the interval [−0.1, 0.1]. Meanwhile, we have the values range in [−0.01,0.01] in 7,247 scenarios (72.47%). Then, the means and variances of all control parameters are varied to perform more experiments. Among over 90% scenarios in each experiment, the absolute values of $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ are less than 0.1. Based on the DOE theory, the effective sample size for the test is 9,025. The simulation adopts 10,000 scenarios; thus, the finding is effective in the two-schedule problem. Fig. 4.

Experiment 2. Correlative Demands with white Gaussian Noise

In Experiment 1, we assume DS and DF demands are time-dependent, where they follow normal distribution with colored Gaussian noise. Experiment 1 drives to two observations, we further examine their robustness under uncertain demand with white Gaussian noise by Experiment 2 (Appendix 2). The main experimental results in both experiments are presented in Table 2.

We find the main observations in Experiment 1 still hold in the second one. Observations 1 and 2 are verified in the experiment under simple decision circumstance with two allocation schedule. We also examine the two observations under different decision circumstance with more allocation schedules and parameter scopes. The optimal quantities of the empty container under the dynamic policy are all smaller than the quantities under the myopic policy in all scenarios. Further, the optimal empty container quantity under the dynamic policy is almost identical to the static policy in most situations. The observations are prohibited impossible to analytically prove since stochastic dynamic programming is one of NP-hard problems, but the observations are examined by two numerical experiments above.

As presented above, analytically solving a stochastic dynamic programming is hard and large-scale simulations are often employed to have the numerical solutions. The allocation decision at each schedule is a dynamic programming model that considers all future demands as unrealized random variables, numerically solving the stochastic dynamic programming by simulations are also burdensome. For example, given a *T*-schedule allocation framework, we must evaluate almost *T* dynamic stochastic programs to obtain the optimal empty container quantity. Thus, finding the optimal solution represents a significant computational burden. Considering that the optimal empty container quantity from the myopic allocation policy is smaller than the dynamic allocation policy, Observation 1 becomes very useful in reducing the computation iterations to optimize the solution of the dynamic model. As well, if we search optimal solutions by having the static solution as the initial inputs, many computation iterations also can be reduced. In next subsection, a simplified search algorithm to numerically solve the target problem Eqs. (3)–(10) is designed based on the two observations.

4.2. Model solution

(1) **Allocation decisions at current schedule**

The allocation decisions using the dynamic allocation policy are made based on the current empty container quantities and current and future DS and DF. The optimal allocation decision can be generated using the Monte Carlo simulation. In the simulation, the demand values are given by stochastic generation; thus, $v_F d_F^t$ and $v_S d_S^t$ in Eq. (6) are known values and are ineffective to the solution. Therefore, the allocation solution in each schedule becomes a static allocation problem, which can be solved using the transportation problem method. The transportation problem-solution framework at schedule *t* is shown in Table 3.

The optimal transportation quantities $Y^t = \{y_F^{D\_t}, y_S^{D\_t}\}$ corresponding to demands $D^t = \{d_F^t, d_S^t\}$ are the optimal empty container-allocation quantities at schedule *t*.

(2) **Search algorithm for the optimal empty container quantity**

Considering that $\Pi^D(Q_0)$ is concave in Q_0^D (Lemma 1), optimal empty container quantity Q_0^{D*} exists to maximize $\Pi^D(Q_0^{D*})$. Finding the optimal solution is a significant computational burden; thus, we design a search algorithm for the dynamic model. Based on Observation 2, we consider the optimal empty container quantity Q_0^{S*} from the static allocation policy because the initial value can always reduce the calculation iterations for optimal quantity Q_0^{D*}. The algorithm is designed as follows (Fig. 5).

The basic processes of the algorithm for the dynamic model are as follows:

Table 2
Experiment results in both experiments.

	Percentage of $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*} \in [0.1, 0.1]$	Percentage of $Q_0^{M*} - Q_0^{D*} \geq 0$
Experiment 1 with Colored Gaussian Noise	91.83%	100%
Experiment 2 with White Gaussian Noise	94.66%	100%

Table 3
Transportation problem at schedule *t*.

Current inventory	Realized demand at schedule *t*		Expected demand in future schedules					Residual
	d_F^t	d_S^t	D_F^{t+1}	D_S^{t+1}	...	D_F^T	D_S^T	R^T
x_F^t	$-\alpha_F$	$+\infty$	$-\alpha_F$	$+\infty$	...	$-\alpha_F$	$+\infty$	0
$X^{D\_t} - x_F^t$	$+\infty$	$-\alpha_S$	$+\infty$	$-\alpha_S$	...	$+\infty$	$-\alpha_S$	0

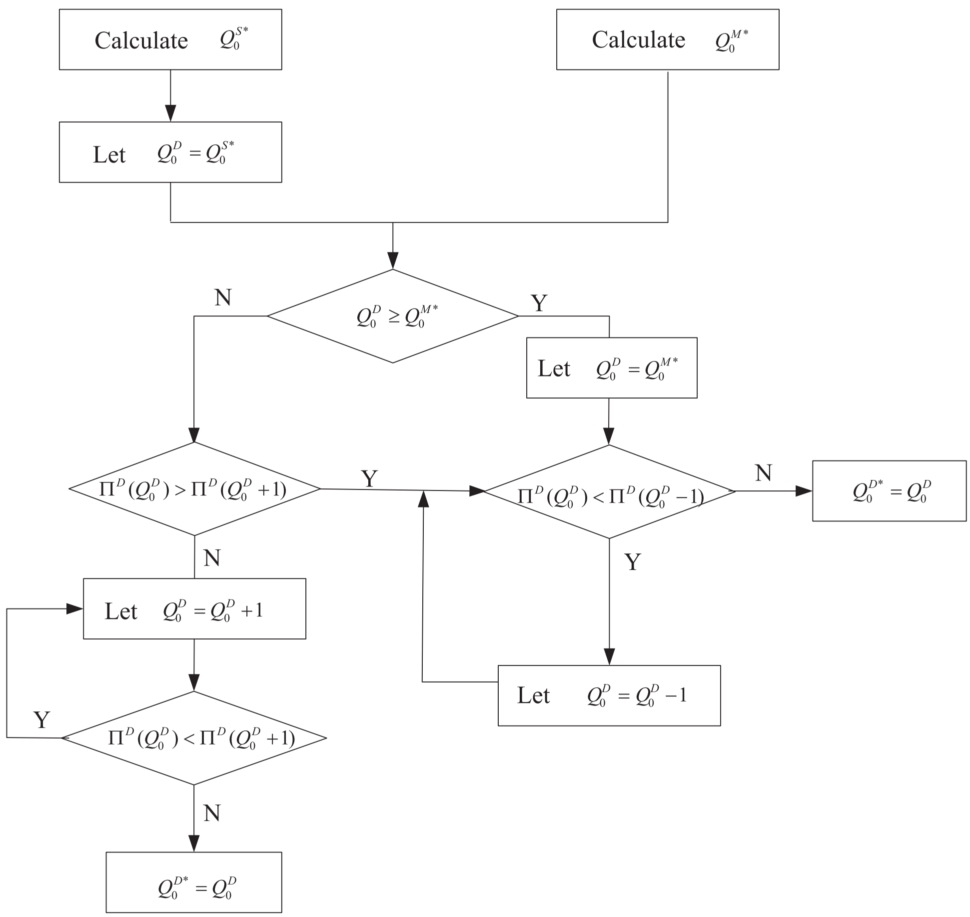

Fig. 5. Algorithm for the dynamic allocation policy.

Step 1. Solve the problem by using the static allocation policy, and obtain the optimal empty container quantity Q_0^{S*}. Solve the problem by using the myopic allocation policy, and obtain the optimal empty container quantity Q_0^{M*}. Let $Q_0^D = Q_0^{S*}$.
Step 2. If $Q_0^D \geq Q_0^{M*}$, go to Step 4; else, go to Step 3.
Step 3. Calculate $\Pi^D(Q_0^D)$ and $\Pi^D(Q_0^D + 1)$. If $\Pi^D(Q_0^D) > \Pi^D(Q_0^D + 1)$, go to Step 4; else, go to
Step 6.
Step 4. Calculate $\Pi^D(Q_0^D)$ and $\Pi^D(Q_0^D - 1)$, then go to Step 5.
Step 5. If $\Pi^D(Q_0^D) < \Pi^D(Q_0^D - 1)$, let $Q_0^D = Q_0^D - 1$ and go to Step 4; else $Q_0^{D*} = Q_0^D$.
Step 6. Let $Q_0^D = Q_0^D + 1$, and calculate $\Pi^D(Q_0^D)$ and $\Pi^D(Q_0^D + 1)$, then go to Step 7.
Step 7. If $\Pi^D(Q_0^D) < \Pi^D(Q_0^D + 1)$, go to Step 6; else $Q_0^{D*} = Q_0^D$.

4.3. Numerical example

To illustrate the effectiveness of the proposed algorithm, a numerical experiment with two allocation schedules is implemented. All demands follow the normal distributions truncated at zero and rounded to the nearest integer. The given parameters are $d_F^1 \sim N(43, 36)$, $d_F^2 \sim N(43, 36)$, $d_S^1 \sim N(24, 17)$, $d_S^2 \sim N(24, 17)$, $c_0 = 7$, $K_0 = 15$, $v_F = 18$, $p_F = 30$, $u_F = 15$, $v_S = 2$, $p_S = 36$, $u_S = 12$, and $w = 1$. We first compute the optimal empty container quantity by using the myopic allocation and the static policies. The optimal values are $Q_0^{S*} = 86$ and $Q_0^{M*} = 93$, respectively.

Then, we solve the dynamic model by using our algorithm (Section 4.2). Taking $Q_0^D = Q_0^{S*} = 86$ as the initial value of the dynamic model, the corresponding initial profit is $\Pi^D(86) = 2665$. We set the input value $Q_0^D = 87$ because $87 < Q_0^{M*} = 93$

Table 4
Calculations using our algorithm.

Optimal solution by static and myopic policy	Solution by dynamic policy	Calculation 1	Calculation 2	Calculation 3
$Q_0^{S*} = 86$	Q_0^D	86	**87**	88
$Q_0^{M*} = 93$	$\Pi^D(Q_0^*)$	2665	**3823**	3770

and calculate the corresponding profit $\Pi^D(87) = 3823$. The optimal rented empty container quantity is more than 86 because $\Pi^D(86) < \Pi^D(87)$. We then set the input value $Q_0^D = 88$ because $88 < Q_0^{M*} = 93$ and calculate the corresponding profit $\Pi^D(88) = 3770$. $\Pi^D(86) < \Pi^D(87)$ and $\Pi^D(87) > \Pi^D(88)$; thus, the optimal rented empty container quantity for the carrier is $Q_0^{D*} = 87$. The maximum profit for the carrier is $\Pi^{D*}(Q_0^D) = \Pi^D(Q_0^{D*}) = 3823$. The calculations using our algorithm are shown in Table 4.

In Table 4, the algorithm takes only 3 iterations to obtain the optimal rented empty container quantity, whereas it takes 87 iterations using the standard search method. This finding shows more than 96% of the total calculation burden is saved by using our new searching method. Using the traditional searching method, the initial value of Q_0^D is considered as zero. However, based on our findings, the optimal value of the container quantity using the dynamic policy is almost identical to the value in using the static policy and to the lower limit in using the myopic allocation policy. The numerical example shows that our algorithm is very efficient.

We also perform other experiments to verify the efficiency of our algorithm. For example, we change the distributions of the random variables and the values of the constant parameters, and we find that the proposed algorithm in our paper needs fewer calculations. Then, we perform experiments in relatively complex integrated systems (systems with three and five schedules), which resulted in the need for fewer calculations. The proportions of the reduced calculation burden saved vary from experiment to experiment, but the larger of the number of schedules, the larger will be the saved proportions of the calculations.

5. Conclusions

In this paper, we studied an empty container-allocation system in which the demands arrive over discrete time schedules. The carrier invests in empty containers before the actual demands are known. The carrier faces both DF and DS. Considering the long-term contract between the carrier and the forwarder, the service price of the forwarder is cheaper than that of the spot market. The demands are random and follow certain distributions. The objective is to determine the optimal empty container quantity to maximize the total profit of the carrier. The problem can be divided into two stages, namely, the empty container-preparation and the allocation. The problem can be modeled as a stochastic dynamic programming model because the demands in each schedule are random.

The allocation policy has a direct effect on the optimal empty container-capacity plan. The dynamic allocation policy is proven to be the optimal allocation policy, which satisfies first the DF at the current schedule as much as possible, and satisfies the DS at the current schedule based on the evaluation of future demands. We prove that the objective function of the stochastic dynamic model is concave in empty container quantity, and a single optimal value of the variable exists. Two related similar allocation policies, which are static and myopic, are available. We prove that the dynamic policy is more profitable than the myopic policy. We also find that the optimal empty container quantity allocated by the static policy is almost identical to the dynamic policy. The calculation of the model using the dynamic policy is burdensome; hence, this result is useful in reducing the computational burden of finding the optimal solution. Thus, taking the optimal solution using the static policy as the initial value, a search algorithm is designed for the objective model. The numerical experiments show that the proposed algorithm is effective and requires fewer computations.

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Appendix 1. Dynamic policy is more profitable than myopic policy

Proof. The carrier fixes the container capacity at the preparation stage, then, the number of containers that can be allocated to satisfy the demands in T allocation schedules will be known. Let the available empty containers at the beginning of schedule t be X^t. The satisfied DS using the myopic policy is $y_S^{M\_t} = \min(N^t, d_S^t)$, whereas the satisfied DS using the dynamic policy based on the current and future demands is limited in the range $[0, \min(N^t, d_S^t)]$.

Let ρ be the quantity difference of the satisfied DS using the myopic and dynamic policies at schedule t, that is, $\rho = y_S^{M\_t} - y_S^{D\_t} \geq 0$. Simultaneously, the allocation quantities to the DF at schedule t using the two policies are y_F^{M-t} and $y_F^{D\_t}$, respectively. At schedule T, the dynamic policy does not consider future demands. Thus, the allocated quantities to the DF and DS in this schedule are the same under the dynamic and myopic policies,

$$y_F^{D\_T} = y_F^{M\_T},\ y_S^{D\_T} = y_S^{M\_T} \tag{22}$$

and

$$\Pi^{M\_T} = y_F^{M\_T}\alpha_F + y_S^{M\_T}\alpha_S = y_F^{D\_T}\alpha_F + y_S^{D\_T}\alpha_S = \Pi^{D\_T} \tag{23}$$

Obviously, $\Pi^{M\_t+1} \leq \Pi^{D\_t+1}$. Given the same initial empty container quantity S^t, because of Eq. (17) and the fact that $\Pi^{M\_t+1} \leq \Pi^{D\_t+1}$, the profit of the firm from the t^{th} schedule to the T^{th} schedule under myopic policy Π^{M_t} can be calculated

$$\begin{aligned}
\Pi^{M_t}(S^t) &= y_F^{M_t}\alpha_F + y_S^{M_t}\alpha_S + \Pi^{M_t+1}(S^t - y_F^{M_t} - y_S^{M_t}) \\
&= y_F^{D_t}\alpha_F + (y_S^{D_t} + \rho)\alpha_S + \Pi^{M_t+1}(S^t - y_F^{D_t} - y_S^{D_t} - \rho) \\
&\leq y_F^{D_t}\alpha_F + (y_S^{D_t} + \rho)\alpha_S + \Pi^{D_t+1}(S^t - y_F^{D_t} - y_S^{D_t} - \rho) \\
&= y_F^{D_t}\alpha_F + y_S^{D_t}\alpha_S + (\rho\alpha_S + \Pi^{D_t+1}(S^t - y_F^{D_t} - y_S^{D_t} - \rho)) \\
&\leq y_F^{D_t}\alpha_F + y_S^{D_t}\alpha_S + \Pi^{D_t+1}(S^t - y_F^{D_t} - y_S^{D_t}) \\
&= \Pi^{D_t}(S^t)
\end{aligned} \tag{24}$$

On the right-hand side of Eq. (24), the first and second terms are the revenues for satisfying the DS and DF at schedule t, respectively, whereas the third term is the profit generated from $(t+1)^{th}$ to $T^{\,th}$ schedule. From Eq. (24), we have $\Pi^{D\_t} \geq \Pi^{M\_t}$, which means that the dynamic policy is more profitable than the myopic policy given the same initial empty container quantities, *Q.E.D.*

Appendix 2. Design and observations of Experiment 2

In Experiment 2, we assume the uncertain demands follow white Gaussian noise, which means DF demand and DS demand

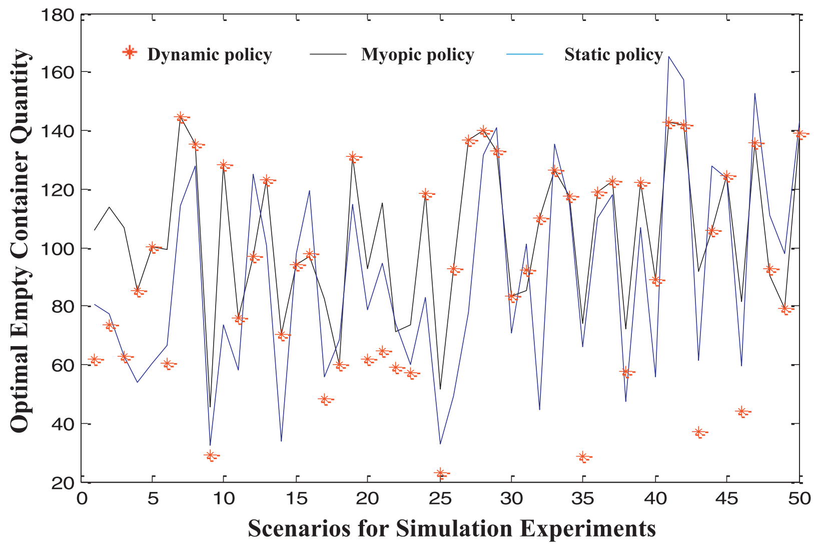

Fig. A1. Optimal empty container quantity by different allocation policies.

are realized under the same normal distribution in different allocation periods, respectively. In the Experiment 2 with two schedules, we assume that all demands follow normal distributions (truncated at zero and rounded to the nearest integer). The penalty cost, carrier-owned container quantity, and empty-container preparation cost follow normal distribution (truncated at zero and positive). The parameter assumptions are as follows: $d_F^1 \sim n(30, 16)$, $d_F^2 \sim n(30, 16)$, $d_S^1 \sim n(28, 12)$, $d_S^2 \sim n(28, 12)$, $c_0 \sim n(6, 12)$, $v_1 \sim n(6, 11)$, $p_1 \sim n(30, 11)$, $u_1 \sim n(7, 10)$, $K_0 \sim n(10, 16)$, and $w = p_1 B(3, 5)$, where n denotes normal distribution and B denotes beta distribution.

The revenue for satisfying one DF unit is $\alpha_F = p_1 + v_1 - u_1 - w$. According to both real practice and our assumptions, $\alpha_F > \alpha_S$. Thus, we let $\alpha_S = \alpha_F B(5, 8)$. To perform the comparative study, the values of each parameter in the three allocation policies are assumed as identical. We generate 50 scenarios to compare the optimal empty container quantity by using the different allocation policies (Fig. 2).

Fig. A1 shows the simulation experiment results on the profits from the three allocation policies. The results provide directions for searching a possible solution to the optimal input material quantity of the system. The simulations show that the optimal empty container quantity using the static allocation policy is not always more than the quantity resulting from the myopic policy or lesser than the quantity resulting from the dynamic policy. However, the optimal empty container quantity using the myopic policy is always more than or equal to that using the dynamic policy. To confirm this observation, more scenarios are generated by the DOE method. A two-tailed test statistical experiment is designed, and the corresponding confidence level, test power, and permissible error of the result are 0.95, 0.1, and 1, respectively. We generated 5,000 random scenarios such that more extreme values and parameter combinations are included in the experiment. The mean of the $Q_0^{M*} - Q_0^{D*}$ values is 0.0313, and the standard variance is 0.1377; hence, the basic effective scenario size for the statistical evaluation is just 2,464 by using the DOE theory. In other words, we generate an adequate number of scenarios to perform the experiment effectively. The $Q_0^{M*} - Q_0^{D*}$ values for each scenario are plotted in Fig. A2.

Then, we obtain the values of $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ and $(Q_0^{M*} - Q_0^{D*})/Q_0^{D*}$ from the 5,000 simulation scenarios (Fig. A3). From the statistics of the scenarios, 4,733 scenarios (approximately 94.66%)

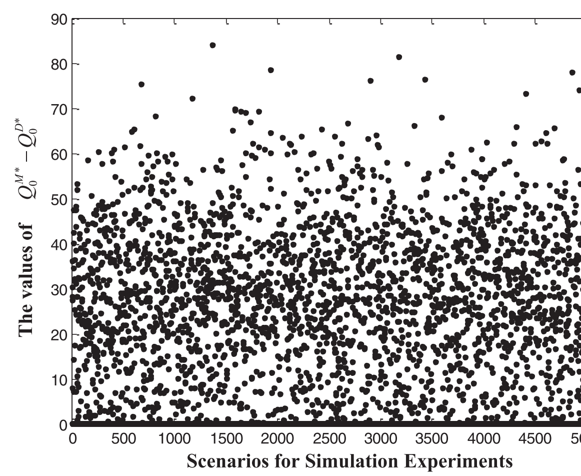

Fig. A2. Comparison of optimal empty container quantities using the myopic and dynamic policies with big sample sizes.

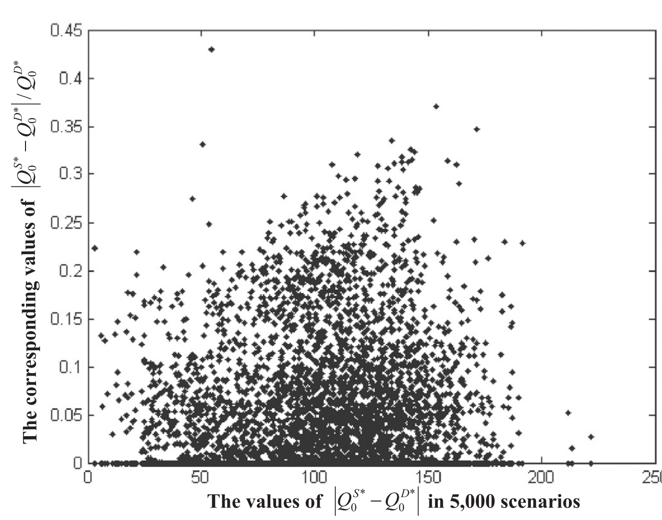

Fig. A3. $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ values in 5,000 simulations.

have $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ values within the interval [−0.1, 0.1]. Meanwhile, 3,891 scenarios (76.82%) have zero values. Then, the means and variances of all control parameters are varied to perform more experiments. Among over 90% scenarios in each experiment, the absolute values of $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ are less than 0.1. Based on the DOE theory, the effective sample size for the test is 4,273. The simulation adopts 5,000 scenarios; thus, the finding is effective in the two-schedule simulation framework. However, when we test the $(Q_0^{M*} - Q_0^{D*})/Q_0^{D*}$ values for 5,000 scenarios, only 42.38% have corresponding values within the interval [−0.1, 0.1].

We also examine Observations 1 and 2 under different experiments with multi-schedules. In all scenarios, the optimal quantities of the empty container under the dynamic policy are all smaller than the quantities under the myopic policy. Further, the optimal empty container quantity under the dynamic policy is always identical to the static policy. In most scenarios, the difference rates $(Q_0^{S*} - Q_0^{D*})/Q_0^{D*}$ are included within the interval [0.15, 0.15].

The allocation decision at each schedule is a dynamic programming model that considers all future demands as unrealized random variables. For example, given a *T*-schedule allocation framework, we must evaluate almost *T* dynamic stochastic programs to obtain the optimal empty container quantity. Thus, finding the optimal solution represents a significant computational burden. Considering that the optimal empty container quantity from the myopic allocation policy is smaller than the dynamic allocation policy, Observation 2 becomes very useful in reducing the computation iterations to optimize the solution of the dynamic model.

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